Lattices

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An example of J. Martinet (L23aa)

First we need to solve the equation $\sum_{k=1}^{n} k^2 = m^2$

```
? hyperellratpoints(x*(x+1)*(2*x+1)*6,[10^7,1])
%1 = [[-1,0],[0,0],[1,6],[1,-6],[24,420],[24,-420]]
? sum(k=1,24,k^2)
%2 = 4900
```

We compute the orthogonal of the vector $[2, 3, \dots, 24, 70]$.

```
? V=matkerint(Mat(concat([2..24],70)));
```

We build a dimension-23 lattice as follows:

```
? M=matrix(23,23,i,j,V[,i]~*V[,j]-2*V[24,i]*V[24,j]
? Q=qfminim(M);Q[1..2]
%6 = [4600,3];
```

So this lattice has 4600 minimal vectors of norm 3.

Automorphisms

```
? G=qfauto(M);
? G[1]
%7 = 84610842624000
? #G[2]
%8 = 3
? #qforbits(G,Q)
%9 = 1
```

So the automorphism group has 84610842624000 elements, is given by 3 generators and acts transitively on the minimal vectors (`qfautoexport` allow to export it to GAP).

Modular form

We define the theta function of the associated even lattice:

```
? [mf, F, v] = mffromqf(2*M);
? mfparams(mf)
%11 = [4,23/2,1,4,t-1]
? mfcoefs(F,10)
%12 = [1,0,0,4600,93150,953856,6476800,32788800,133
```

Note that computing a_{10} by enumeration would take a quite long time.

Madelung constant

The Madelung constant is the limit $\lim_{n\mapsto\infty}\sum_{(x,y,z)\in\mathbb{Z}^3-\{0,0,0\},max(|x|,|y|,|z|)\leq n}\frac{(-1)^{x+y+z}}{\sqrt{x^2+y^2+z^2}}$. One way to compute it

```
? L1=lfunqf(matdiagonal([1,1,1]));
? L2=lfunqf(matdiagonal([4,1,1]));
? L3=lfunqf(matdiagonal([4,4,1]));
? 3*lfun(L1,1/2)+6*lfun(L2,1/2)-12*lfun(L3,1/2)
%16 = -1.7475645946331821906362120355443974035
```