Defining L-functions in GP

A tutorial

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Riemann ζ function

```
? Zeta = lfuncreate(1)
%1 = [[Vecsmall([1]),1],0,[0],1,1,1,1]
? lfun(Zeta,2)
%2 = 1.6449340668482264364724151666460251892
? lfun(Zeta,0,1)
%3 = -0.91893853320467274178032973640561763986
? lfun(Zeta,1)
%4 = 1.0000000000*x^-1+O(x^0)
? lfun(Zeta,1+x+O(x^10))
%5 = 1.000000000*x^-1+0.5772156+0.0728158*x-0.00484
? lfunzeros(Zeta,20)
%6 = [14.134725141734693790457251983562470271]
? lfunlambda(Zeta,2)
%7 = 0.52359877559829887307710723054658381403
```

Dirichlet L functions

```
? G=znstar(4,1); G.clgp
%8 = [2, [2], [3]]
? Dir=lfuncreate([G,[1]]); Dir[2..5]
%9 = [0,[1],1,4]
? lfunan(Dir,30)
%10 = [1,0,-1,0,1,0,-1,0,1,0,-1,0,1,0,-1,0,1,0,-1,0
? lfun(Dir,2)
%11 = 0.91596559417721901505460351493238411078
? Catalan
%12 = 0.91596559417721901505460351493238411077
```

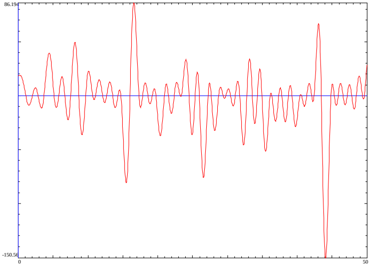
Dirichlet L functions

```
? G=znstar(1001,1); G.clgp
%13 = [720,[60,6,2]]
? chi = [1,0,1];
? Dir1001=lfuncreate([G,chi]); Dir1001[2..5]
%15 = [1,[0],1,1001]
? lfunan(Dir1001,10)
%16 = [1,0.406736643-0.913545458*I,-0.309016994-0.9
? abs(%)
%17 = [1,1.00000000,1.00000000,1.00000000,1.0000000
```

Initialization

When several values of a L function are needed, it is better to first initialize the function on a domain (here $\Re(s) = 1/2$, $|\Im(s)| \leq 50$).

```
? L = lfuninit(Dir1001,[1/2,0,50]);
? ploth(x=0,50,lfunhardy(L,x));
```



Dedekind ζ function

```
? Dedek = lfuncreate(x^2+1); Dedek[2..5]
%20 = [0,[0,1],1,4]
? lfun(Dedek,2)
%21 = 1.5067030099229850308865650481820713960
? zeta(2)*Catalan
%22 = 1.5067030099229850308865650481820713960
? L=lfunmul(Zeta,Dir);
? lfun(L,2)
%24 = 1.5067030099229850308865650481820713960
? L2=lfundiv(Dedek,1);
? lfun(L2,2)
%26 = 0.91596559417721901505460351493238411078
```

Number field

To initialize a number field, use `bnfinit`:

```
? K = bnfinit(x^3-21*x+35);
? K.pol
%28 = x^3-21*x+35
? K.sign
%29 = [3,0]
? K.disc
%30 = 3969
? K.no
%31 = 3
? K.reg
%32 = 4.2016900422410957093939568266541073391
```

Analytic class number formula

```
? L = lfuncreate(K); L[2..5]
%33 = [0,[0,0,0],1,3969]
? lfun(L,1+x+O(x^2))
%34 = 0.800321912807827754170277490791258540 79*x^-1
? {2^K.r1*(2*Pi)^K.r2*K.no*K.reg/
    (K.tu[1]*sqrt(abs(K.disc)))}
%35 = 0.80032191280782775417027749079125854078
? lfun(L,0,2)/2!
%36 = -6.3025350633616435640909352399811610087
? -K.no*K.reg/2
%37 = -6.3025350633616435640909352399811610087
```

Special values

```
? bestappr(lfun(L,-5))
%38 = -23236371792424 3/63
? lfun(L,-11)
%39 = 22746539747839269298470894321688 10063.4
? bestappr(lfun(L,-11))
%40 = 7793522416802062653515308705900939204243 73675
? \p100
   realprecision = 115 significant digits (100 digi
? bestappr(lfun(L,-11))
%41 = 1862941605348036155544766244946255441912 1/819
```