

DESCRIPTION AND USE OF THE TABLES

For the number fields belonging to tables of reasonable length available to us, we have computed the signature, the Galois group of the Galois closure of the field, the discriminant of the number field and the index of $\mathbb{Z}[x]$ in the ring of integers, the class number, the structure of the class group as a product of cyclic groups, an ideal in the class for each class generating these cyclic groups, the regulator, the number of roots of unity in the field, a generator of the torsion part of the unit group and a system of fundamental units.

The computations have been done using the PARI package, which assumes GRH.

All these tables are available by anonymous ftp from

`megrez.math.u-bordeaux.fr` (147.210.16.17)

This directory contains subdirectories with files corresponding to tables of number fields which have been compiled by different authors. The tables under consideration are tables of degrees 3, 4, 5, 6 and 7 for all possible signatures.

The length of the tables is rather small for degrees 6 and 7.

Each directory contains indexes corresponding to each signature, so that it is easy to determine in which table a given number field belongs (there may be an ambiguity if the same discriminant appears in two consecutive tables).

For the tables considered, the size and the name of the authors of the computations are as follows:

**	degree= 3	signature= (1, 1)	$ d_K \leq 1\,000\,000$	by M. Olivier
**	degree= 3	signature= (3, 0)	$ d_K \leq 2\,000\,000$	by M. Olivier
**	degree= 4	signature= (0, 2)	$ d_K \leq 1\,000\,000$	by J. Buchmann, D. Ford, M. Pohst
**	degree= 4	signature= (2, 1)	$ d_K \leq 1\,000\,000$	by J. Buchmann, D. Ford, M. Pohst
**	degree= 4	signature= (4, 0)	$ d_K \leq 1\,000\,000$	by J. Buchmann, D. Ford, M. Pohst
**	degree= 5	signature= (1, 2)	$ d_K \leq 1\,000\,000$	by F. Diaz y Diaz, M. Pohst, A. Schwarz
**	degree= 5	signature= (3, 1)	$ d_K \leq 1\,000\,000$	by F. Diaz y Diaz, M. Pohst, A. Schwarz
**	degree= 5	signature= (5, 0)	$ d_K \leq 20\,000\,000$	by F. Diaz y Diaz, M. Pohst, A. Schwarz
**	degree= 6	signature= (0, 3)	$ d_K \leq 200\,000$	by M. Olivier
**	degree= 6	signature= (2, 2)	$ d_K \leq 400\,000$	by M. Olivier
**	degree= 6	signature= (4, 1)	$ d_K \leq 1\,000\,000$	by M. Olivier
**	degree= 6	signature= (6, 0)	$ d_K \leq 10\,000\,000$	by M. Olivier
**	degree= 7	signature= (1, 3)	$ d_K \leq 600\,000$	by P. Létard
**	degree= 7	signature= (3, 2)	$ d_K \leq 1\,800\,000$	by P. Létard
**	degree= 7	signature= (5, 1)	$ d_K \leq 12\,000\,000$	by P. Létard
**	degree= 7	signature= (7, 0)	$ d_K \leq 150\,000\,000$	by P. Létard

The files containing the arithmetic information are denoted by $txy.zzz$ where x is a digit that indicates the degree of the number field in the table, y is a digit corresponding to the number of real places of the number field in the table and zzz is a three digits number that denotes the order number of the file in the txy -table when the corresponding table contains more than 1000 discriminants. Hence, the length of $txy.zzz$ is exactly equal to 1000 except for the largest value of zzz in a given signature. In this last case the length is always less than 1000.

Each file contains a single Pari vector having 1000 components (except for the last which may contain less), each component corresponding to a single number field. After being gzipped, they are human-readable, but are also made to be read by GP. Since the file size is much larger than the default GP buffer (30000), and the necessary stack size is also insufficient, one should use the following command to launch GP:

```
gp -s 10000000 -b 500000
```

and then use the usual GP command `\r filename` to read in the 1000 (or less) number fields as a single vector v .

The information contained in each one of these components appears as a new vector having 9 components and the following structure :

Let k , $1 \leq k \leq m$ be fixed. Then we have

$v[k][1] = P(x)$	the polynomial generating the field (1)
$v[k][2] = [r_1, r_2]$	the signature of the field (2)
$v[k][3] = [\#G, s, m]$	the Galois group G of $P(x)$ (3)
$v[k][4] = [d, a]$	the discriminant of the field and the index of $[\mathbb{Z}_K : \mathbb{Z}[x]]$
$v[k][5] = [1, w_2, \dots, w_n]$	an integral basis given in HNF on the power basis
$v[k][6] = [h, [c_1, \dots, c_t], [\mathfrak{a}_1, \dots, \mathfrak{a}_t], [\alpha_1, \dots, \alpha_t]]$	the class number, the structure of the class group as a product of cyclic groups of orders c_1, \dots, c_t , an ideal generating these cyclic groups and the generator of the principal ideal $\mathfrak{a}_i^{c_i}$ (4)
$v[k][7] = R$	the regulator
$v[k][8] = [w, \zeta]$	the number of roots of unity in the field and a generator of this cyclic group (5)
$v[k][9] = [u_1, \dots, u_r]$	a system of fundamental units (6)

(1) The polynomial used to define a number field is not completely canonical, but can be obtained via a completely deterministic process: If $P(X) = a_n X^n + \dots + a_0$, we set $\text{size}(P) = \sum_i |\theta_i|^2$, where the θ_i are the complex roots of P . Then the polynomial P which is chosen is a *monic* polynomial which minimizes for the lexicographic order the vector

$v(P) = (\text{size}(P), |\text{disc}(P)|, |a_n|, \dots, |a_0|)$ and such that the non-zero monomial of largest degree d such that $d \not\equiv n \pmod{2}$, is one exists, is negative.

It is possible that this still does not determine the polynomial P completely, but in the range of our tables it does.

Note that the choice of P is slightly different from the one made in [Cohen] (p. 170-171), in particular because of the exchange of the $\text{size}(P)$ and $|\text{disc}(P)|$ components. Indeed, making a complete list of polynomials P having the smallest possible $\text{size}(P)$ is a straightforward backtracking procedure, and in this small finite list one can then choose the smallest lexicographic polynomial. On the other hand, finding the smallest possible discriminants involve finding all the solutions of index norm equations, and this is a difficult process.

(2) As usual, r_1 denotes the number of real places of K and r_2 the number of pairs of complex places.

(3) The Galois group G of $P(x)$ is given as a three-component vector. The first component gives the order of the group, the second component denotes the group signature (hence, $s = 1$ if $G \subset A_n$ and $s = -1$ otherwise) and the third component denotes the number of the groups corresponding to the same pairs $(\#G, s)$. For the tables considered here, one has $m = 1$ except in two cases in degree six:

$$C_6 = [6, -1, 1], S_3 = [6, -1, 2], S_4^- = [24, -1, 1], A_4 \times C_2 = [24, -1, 2].$$

(4) The first component of this vector denotes the class number. When one has $h = 1$, the other three components are empty vectors. The second component is a vector $[c_1, \dots, c_t]$ giving the structure of the class group as a product of cyclic groups:

$$\mathcal{H}_K \approx \mathbb{Z}/c_1\mathbb{Z} \times \dots \times \mathbb{Z}/c_t\mathbb{Z}$$

with $c_i | c_{i-1}$. The third component is an other vector having t components each one of them gives an ideal in a class generating the corresponding cyclic group. Finally, the last component is a vector whose i th-component is a generator of the principal ideal $\mathfrak{a}_i^{c_i}$.

Note that the ideals used as generators of the cyclic factors are not canonical, and we are aware that it is easy to find much smaller ideals in many cases.

(5) The first component of this vector denotes the order of the group of roots of unity into the field and the second component gives a generator of this cyclic group.

(6) As usual, one has $r = r_1 + r_2 - 1$ for the rank of the unit group.

All the elements of the field are given as vectors in the integral basis. All the ideals are given by their HNF matrix in the integral basis.

Evidently we welcome corrections to the tables. We also welcome additional contributions. However, in that case we ask you to use exactly the same format for submitting your tables. If you want to submit only the equations for the number fields, we can compute the necessary invariants. Note that additional data is most welcome in tables where only a small number of fields is available (e.g. not in degree 3 or 4).