

Pari-GP reference card

(PARI-GP version 2.16.2)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: **gp**
To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function *?function*
extended description *??keyword*
list of relevant help topics *???pattern*
name of GP-1.39 function *f* in GP-2.* *whatnow(f)*

Input/Output

previous result, the result before *%, %^, %^-*, etc.
n-th result since startup *%n*
separate multiple statements on line *;*
extend statement on additional lines **
extend statements on several lines *{seq1; seq2;}*
comment */* ... */*
one-line comment, rest of line ignored *\\ ...*

Metacommands & Defaults

set default *d* to *val* *default({d},{val})*
toggle timer on/off *#*
print time for last result *##*
print defaults *\d*
set debug level to *n* *\g n*
set memory debug level to *n* *\gm n*
set *n* significant digits / bits *\p n, \pb n*
set *n* terms in series *\ps n*
quit GP *\q*
print the list of PARI types *\t*
print the list of user-defined functions *\u*
read file into GP *\r filename*
set debuglevel for domain *D* to *n* *setdebug(D,n)*

Debugger / break loop

get out of break loop *break* or **<C-D>**
go up/down *n* frames *dbg_up({n}), dbg_down*
set break point *breakpoint()*
examine object *o* *dbg_x(o)*
current error data *dbg_err()*
number of objects on heap and their size *getheap()*
total size of objects on PARI stack *getstack()*

PARI Types & Input Formats

t_INT. Integers; hex, binary *±31; ±0x1F, ±0b101*
t_REAL. Reals *±3.14, 6.022 E23*
t_INTMOD. Integers modulo *m* *Mod(n,m)*
t_FRAC. Rational Numbers *n/m*
t_FFELT. Elt in finite field **F_q** *ffgen(q,'t)*
t_COMPLEX. Complex Numbers *x + y * I*
t_PADIC. *p*-adic Numbers *x + 0(p^k)*
t_QUAD. Quadratic Numbers *x + y * quadgen(D,{'w})*
t_POLMOD. Polynomials modulo *g* *Mod(f,g)*
t_POL. Polynomials *a * x^n + ... + b*
t_SER. Power Series *f + 0(x^k)*
t_RFRAC. Rational Functions *f/g*
t_QFB. Binary quadratic form *Qfb(a,b,c)*
t_VEC/t_COL. Row/Column Vectors *[x,y,z], [x,y,z]~*
t_VEC integer range *[1..10]*

t_VECSMALL. Vector of small ints *Vecsmall([x,y,z])*
t_MAT. Matrices *[a,b;c,d]*
t_LIST. Lists *List([x,y,z])*
t_STR. Strings *"abc"*
t_INFINITY. $±∞$ *+oo, -oo*

Reserved Variable Names

$π ≈ 3.14, γ ≈ 0.57, C ≈ 0.91, I = √-1$ **Pi, Euler, Catalan, I**
Landau's big-oh notation **O**

Information about an Object, Precision

PARI type of object *x* *type(x)*
length of *x* / size of *x* in memory *#x, sizebyte(x)*
real precision / bit precision of *x* *precision(x), bitprecision(x)*
p-adic, series prec. of *x* *padicprec(x,p), serprec(x,v)*
current dynamic precision *getlocalprec, getlocalbitprec*

Operators

basic operations *+, -, *, /, ^, sqr*
*i←i+1, i←i-1, i←i*j, ...* *i++, i--, i*=j,...*
Euclidean quotient, remainder *x\y, x%y, x%y, divrem(x,y)*
shift *x* left or right *n* bits *x<<n, x>>n* or *shift(x,±n)*
multiply by 2^n *shiftmul(x,n)*
comparison operators *<=, <, >=, >, ==, !=, ==, lex, cmp*
boolean operators (or, and, not) *||, &&, !*
bit operations *bitand, bitneg, bitor, bitxor, bitneg*
maximum/minimum of *x* and *y* *max(x,y), min(x,y)*
sign of *x* (gives $-1, 0, 1$) *sign(x)*
binary exponent of *x* *exponent(x)*
derivative of *f*, 2nd derivative, etc. *f', f'', ...*
differential operator *diffop(f,v,d,{n=1})*
quote operator (formal variable) *'x*
assignment *x = value*
simultaneous assignment $x ← v[1], y ← v[2]$ *[x,y] = v*

Select Components

Caveat: components start at index $n = 1$.
n-th component of *x* *component(x,n)*
n-th component of vector/list *x* *x[n]*
components *a, a + 1, ... , b* of vector *x* *x[a..b]*
(*m, n*)-th component of matrix *x* *x[m,n]*
row *m* or column *n* of matrix *x* *x[m,], x[,n]*
numerator/denominator of *x* *numerator(x), denominator(x)*

Random Numbers

random integer/prime in $[0, N[$ *random(N), randomprime(N)*
get/set random seed *getrand, setrand(s)*

Conversions

to vector, matrix, vec. of small ints *Col/Vec, Mat, Vecsmall*
to list, set, map, string *List, Set, Map, Str*
create (*x mod y*) *Mod(x,y)*
make *x* a polynomial of *v* *Pol(x,{v})*
variants of **Pol** *et al.*, in reverse order *Polrev, Vecrev, Colrev*
make *x* a power series of *v* *Ser(x,{v})*
convert *x* to simplest possible type *simplify(x)*
object *x* with real precision *n* *precision(x,n)*
object *x* with bit precision *n* *bitprecision(x,n)*
set precision to *p* digits in dynamic scope *localprec(p)*
set precision to *p* bits in dynamic scope *localbitprec(p)*

Character strings

convert to TeX representation *strtex(x)*
string from bytes / from format+args *strchr, sprintf*
split string / join strings *strsplit, strjoin*
convert time *t* ms. to h, m, s, ms format *strtime(t)*

Conjugates and Lifts

conjugate of a number *x* *conj(x)*
norm of *x*, product with conjugate *norm(x)*
 L^p norm of *x* ($L^∞$ if no *p*) *normlp(x,{p})*
square of L^2 norm of *x* *norml2(x)*
lift of *x* from Mods and *p*-adics *lift, centerlift(x)*
recursive lift *liftall*
lift all **t_INT** and **t_PADIC** ($→$ **t_INT**) *liftint*
lift all **t_POLMOD** ($→$ **t_POL**) *lifttpol*

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. **cmp**)
intersection of sets *x* and *y* *setintersect(x,y)*
set of elements in *x* not belonging to *y* *setminus(x,y)*
symmetric difference $xΔy$ *setdelta(x,y)*
union of sets *x* and *y* *setunion(x,y)*
does *y* belong to the set *x* *setsearch(x,y,{flag})*
set of all $f(x,y), x ∈ X, y ∈ Y$ *setbinop(f,X,Y)*
is *x* a set? *setisset(x)*

Lists. create empty list: $L = \text{List}()$
append *x* to list *L* *listput(L,x,{i})*
remove *i*-th component from list *L* *listpop(L,{i})*
insert *x* in list *L* at position *i* *listinsert(L,x,i)*
sort the list *L* in place *listsort(L,{flag})*

Maps. create empty dictionary: $M = \text{Map}()$
attach value *v* to key *k* *mapput(M,k,v)*
recover value attach to key *k* or error *mapget(M,k)*
is key *k* in the dict? (set *v* to $M(k)$) *mapisdefined(M,k,{&v})*
remove *k* from map domain *mapdelete(M,k)*

GP Programming

User functions and closures

x, y are formal parameters; *y* defaults to **Pi** if parameter omitted;
z, t are local variables (lexical scope), *z* initialized to 1.
fun(*x, y*=**Pi**) = **my**(*z*=1, **t**); *seq*
fun = (*x, y*=**Pi**) -> **my**(*z*=1, **t**); *seq*

attach help message *h* to *s* *addhelp(s,h)*
undefine symbol *s* (also kills help) *kill(s)*
Control Statements (*X*: formal parameter in expression *seq*)
if $a ≠ 0$, evaluate *seq1*, else *seq2* *if(a,{seq1},{seq2})*
eval. *seq* for $a ≤ X ≤ b$ *for(X = a,b,seq)*
... for $X ∈ v$ *foreach(v,X,seq)*
... for primes $a ≤ X ≤ b$ *forprime(X = a,b,seq)*
... for primes $≡ a \pmod q$ *forprimestep(X = a,b,q,seq)*
... for composites $a ≤ X ≤ b$ *forcomposite(X = a,b,seq)*
... for $a ≤ X ≤ b$ stepping *s* *forstep(X = a,b,s,seq)*
... for *X* dividing *n* *fordiv(n,X,seq)*
... $X = [n, factor(n)], a ≤ n ≤ b$ *forfactored(X = a,b,seq)*
... as above, *n* squarefree *forsquarefree(X = a,b,seq)*
... $X = [d, factor(d)], d | n$ *fordivfactored(n,X,seq)*
multivariable **for**, lex ordering *forvec(X = v,seq)*

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loop over partitions of n
... permutations of S
... subsets of $\{1, \dots, n\}$
... k -subsets of $\{1, \dots, n\}$
... vectors v , $q(v) \leq B$; $q > 0$
... $H < G$ finite abelian group
evaluate seq until $a \neq 0$
while $a \neq 0$, evaluate seq
exit n innermost enclosing loops
start new iteration of n -th enclosing loop
return x from current subroutine

Exceptions, warnings

raise an exception / warning
type of error message E
try seq_1 , evaluate seq_2 on error

Functions with closure arguments / results

number of arguments of f
select from v according to f
apply f to all entries in v
evaluate $f(a_1, \dots, a_n)$
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$
calling function as closure

Sums & Products

sum $X = a$ to $X = b$, initialized at x
sum entries of vector v
product of all vector entries
sum $expr$ over divisors of n
... assuming $expr$ multiplicative
product $a \leq X \leq b$, initialized at x
product over primes $a \leq X \leq b$

Sorting

sort x by k -th component
min. m of x ($m = x[i]$), max.
does y belong to x , sorted wrt. f
 $\prod g^x \rightarrow$ factorization (\Rightarrow sorted, unique g)

Input/Output

print with/without $\backslash n$, TeX format
pretty print matrix
print fields with separator
formatted printing
write $args$ to file
write x in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in r (ead), w (rite) or a (ppend) mode.
get descriptor n for file $path$ in given $mode$
... from shell cmd output (pipe)

close descriptor
commit pending write operations
read logical line from file
... raw line from file
write $s \backslash n$ to file
... write s to file

forpart($p = n, seq$)
forperm(S, p, seq)
forsubset(n, p, seq)
forsubset($[n, k], p, seq$)
forqfvec(v, q, b, seq)
forsubgroup($H = G$)
until(a, seq)
while(a, seq)
break($\{n\}$)
next($\{n\}$)
return($\{x\}$)

error(), warning()
errname(E)
iferr(seq_1, E, seq_2)
arity(f)
select(f, v)
apply(f, v)
call(f, a)
fold(f, a)
self()

sum($X = a, b, expr, \{x\}$)
vecsum(v)
vecprod(v)
sumdiv($n, X, expr$)
sumdivmult($n, X, expr$)
prod($X = a, b, expr, \{x\}$)
prodeuler($X = a, b, expr$)

vecsort($x, \{k\}, \{fl = 0\}$)
vecmin($x, \{\&i\}$), vecmax
vecsearch($x, y, \{f\}$)
matreduce(m)

print, print1, printtex
printp
printsep(sep, \dots), printsep1
printf()
write, write1, writetex($file, args$)
writebin($file, x$)
read($\{file\}$)
readvec($\{file\}$)
readstr($\{file\}$)
input()

fileopen($path, mode$)
fileextern(cmd)
fileclose(n)
fileflush(n)
fileread(n)
filereadstr(n)
filewrite(n, s)
filewrite1(n, s)

Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get \$VAR from environment
expand env. variable in string

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use export for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$
evaluate closures $f[1], \dots, f[n]$
as select
as sum
as vector
eval f for $i = a, \dots, b$
... for each element x in v
... for p prime in $[a, b]$
... for $p = a \bmod q$
... for $i = a, a + s, \dots, b$
... multivariate
export x to parallel world
... all dynamic variables
frees exported value x
... all exported values

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply $matqr$'s transform to v

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$
 $\{x: x \in v \text{ s.t. } f(x)\}$
 $\{g(x): x \in v\}$
row vec. of $expr$ eval'ed at $1 \leq i \leq n$
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$
vector of small ints

gettime()
getabstime()
getwalltime()
alarm($s, expr$)
allocatemem($\{s\}$)
alias(new, old)
install($f, code, \{gpf\}, \{lib\}$)
system(a)
extern(a)
externstr(a)
getenv("VAR")
strexpend(x)

parapply(f, x)
pareval(f)
parselect($f, A, \{flag\}$)
parsum($i = a, b, expr$)
parvector($n, i, \{expr\}$)
parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)
parforeach($v, x, f, \{r\}, \{f_2\}$)
parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)
parforprimestep($p = a, \{b\}, q, f, \{r\}, \{f_2\}$)
parforstep($i = a, \{b\}, s, f, \{r\}, \{f_2\}$)
parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)
export(x)
exportall()
unexport(x)
unexportall()

matsize(x)
 $x * y$
matmultodiagonal(x, y)
concat($x, \{y\}$)
vecextract($x, y, \{z\}$)
 $x \sim$, mattranspose(x)
matadjoint(x)
mateigen(x)
charpoly(x), minpoly(x)
trace(x), matdet(x)
matpermanent(x)
matfrobenius(x)
matqr(x)
mathouseholder(Q, v)

$[g(x) \mid x \leftarrow v, f(x)]$
 $[x \mid x \leftarrow v, f(x)]$
 $[g(x) \mid x \leftarrow v]$
vector($n, \{i\}, \{expr\}$)
vectorv($n, \{i\}, \{expr\}$)
vectorsmall($n, \{i\}, \{expr\}$)

$[c, c \cdot x, \dots, c \cdot x^n]$
 $[1, 2^x, \dots, n^x]$
matrix $1 \leq i \leq m, 1 \leq j \leq n$
define matrix by blocks
diagonal matrix with diagonal x
is x diagonal?
 $x \cdot \text{matdiagonal}(d)$
 $n \times n$ identity matrix
Hessenberg form of square matrix x
 $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$
 $n \times n$ Pascal triangle
companion matrix to polynomial x
Sylvester matrix of x and y

Gaussian elimination

kernel of matrix x
intersection of column spaces of x and y
solve $MX = B$ (M invertible)
one sol of $M * X = B$
basis for image of matrix x
columns of x not in matimage
supplement columns of x to get basis
rows, cols to extract invertible matrix
rank of the matrix x
solve $MX = B \bmod D$
image mod D
kernel mod D
inverse mod D
determinant mod D

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$
evaluate ${}^t x Q x$
signature of quad form ${}^t y * x * y$
decomp into squares of ${}^t y * x * y$
eigenvalues/vectors for real symmetric x

HNF and SNF

upper triangular Hermite Normal Form
HNF of x where d is a multiple of $\det(x)$
multiple of $\det(x)$
HNF of $(x \mid \text{diagonal}(D))$
elementary divisors of x
 q -rank from elementary divisors
elementary divisors of $\mathbf{Z}[a]/(f'(a))$
integer kernel of x
 \mathbf{Z} -module $\leftrightarrow \mathbf{Q}$ -vector space

Lattices

LLL-algorithm applied to columns of x
... for Gram matrix of lattice
find up to m sols of $qf\text{norm}(x, y) \leq b$
 $v, v[i] :=$ number of y s.t. $qf\text{norm}(x, y) = i$
perfection rank of x
find isomorphism between q and Q
precompute for isomorphism test with q
automorphism group of q

Based on an earlier version by Joseph H. Silverman
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powers($x, n, \{c = 1\}$)
dirpowers(n, x)
matrix($m, n, \{i\}, \{j\}, \{expr\}$)
matconcat(B)
matdiagonal(x)
matisdiagonal(x)
matmultiagonal(x, d)
matid(n)
mathess(x)
mathilbert(n)
matpascal($n - 1$)
matcompanion(x)
polsylvestermatrix(x, y)

matker($x, \{flag\}$)
matintersect(x, y)
matsolve(M, B)
matinverseimage(M, B)
matimage(x)
matimagecompl(x)
matsupplement(x)
matindexrank(x)
matrank(x)
matsolvenmod(M, D, B)
matimagemod(M, D)
matkermod(M, D)
matinvmod(M, D)
matdetmod(M, D)

convert `qfauto` for GAP/Magma `qfautoexport(G, {flag})`
 orbits of V under $G \subset \text{GL}(V)$ `qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables `variables()`
 get var. of highest priority (higher than v) `varhigher(name, {v})`
 ... of lowest priority (lower than v) `varlower(name, {v})`

Coefficients, variables and basic operators

degree of f `poldegree(f)`
 coef. of degree n of f , leading coef. `polcoef(f, n)`, `pollead`
 main variable / all variables in f `variable(f)`, `variables(f)`
 replace x by y in f `subst(f, x, y)`
 evaluate f replacing vars by their value `eval(f)`
 replace polynomial expr. $T(x)$ by y in f `substpol(f, T, y)`
 replace x_1, \dots, x_n by y_1, \dots, y_n in f `substvec(f, x, y)`

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ `polrecip(f)`
 gcd of coefficients of f `content(f)`
 derivative of f w.r.t. x `deriv(f, {x})`
 ... n -th derivative of f `derivn(f, n, {x})`
 formal integral of f w.r.t. x `intformal(f, {x})`
 formal sum of f w.r.t. x `sumformal(f, {x})`

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e `polinterpolate(x, {y}, {t}, {&e})`
 monic polynomial from roots r `polfromroots(r)`
 $T_n/U_n, H_n$ `polchebyshev(n)`, `polhermite(n)`
 $P_n, L_n^{(\alpha)}$ `pollegendre(n)`, `pollaguerre(n, a)`
 n -th cyclotomic polynomial Φ_n `polcyclo(n)`
 return n if $f = \Phi_n$, else 0 `poliscyclo(f)`
 is f a product of cyclotomic polynomials? `poliscycloprod(f)`
 Zagier's polynomial of index (n, m) `polzagier(n, m)`

Resultant, elimination

discriminant of polynomial f `poldisc(f)`
 find factors of `poldisc(f)` `poldiscfactors(f)`
 resultant $R = \text{Res}_v(f, g)$ `polresultant(f, g, {v})`
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$ `polresultantext(x, y, {v})`
 solve Thue equation $f(x, y) = a$ `thue(t, a, {sol})`
 initialize t for Thue equation solver `thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of f `polroots(f)`
 bound complex roots of f `polrootsbound(f)`
 number of real roots of f (in $[a, b]$) `polsturm(f, {[a, b]})`
 real roots of f (in $[a, b]$) `polrootsreal(f, {[a, b]})`
 complex embeddings of t .POLMOD z `conjvec(z)`

Roots and Factorization (Finite fields)

factor f mod p , roots `factormod(f, p)`, `polrootsmod`
 factor f over $\mathbf{F}_p[x]/(T)$, roots `factormod(f, [T, p])`, `polrootsmod`
 squarefree factorization of f in $\mathbf{F}_q[x]$ `factormodSQF(f, {D})`
 distinct degree factorization of f in $\mathbf{F}_q[x]$ `factormodDDF(f, {D})`
 factor n -th cyclotomic pol. Φ_n mod p `factormodcyclo(n, p)`

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots `factorpadic(f, p, r)`, `polrootspadic`
 p -adic root of f congruent to a mod p `padicappr(f, a)`
 Newton polygon of f for prime p `newtonpoly(f, p)`
 Hensel lift $A/\text{lc}(A) = \prod_i B[i] \pmod{p^e}$ `polhensellift(A, B, p, e)`
 $T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i)) \in \mathbf{Z}_p[x]$ `polteichmuller(T, p, e)`
 extensions of \mathbf{Q}_p of degree N `padicfields(p, N)`

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(PARI-GP version 2.16.2)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n `polysym(f, n)`
 Graeffe transform of f , $g(x^2) = f(x)f(-x)$ `polgraeffe(f)`
 factor f over coefficient field `factor(f)`
 cyclotomic factors of $f \in \mathbf{Q}[X]$ `polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).
 find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ `ffinit(p, n, {x})`
 Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ `t = ffgen(T, 't)`
 ... indirectly, with implicit T `t = ffgen(q, 't); T = t.mod`
 map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ `m = ffbend(a, b)`
 build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, `ffextend(a, P)`
 evaluate map m on x `ffmap(m, x)`
 inverse map of m `ffinvm(m)`
 compose maps $m \circ n$ `ffcompom(m, n)`
 x as polmod over codomain of map m `ffmaprel(m, x)`
 F^n over $\mathbf{F}_q \ni a$ `fffrobenius(a, n)`
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$ `ffnbirred(q, n)`

Formal & p -adic Series

truncate power series or p -adic number `truncate(x)`
 valuation of x at p `valuation(x, p)`

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x `taylor(f, x)`
 Laurent series of closure F up to x^k `laurentseries(f, k)`
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ `serconvol(a, b)`
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ `serlaplace(f)`
 reverse power series F so $F(f(x)) = x$ `serreverse(f)`
 remove terms of degree $< n$ in f `serchop(f, n)`
 Dirichlet series multiplication / division `dirmul, dirdiv(x, y)`
 Dirichlet Euler product (b terms) `direuler(p = a, b, expr)`

Transcendental and p -adic Functions

real, imaginary part of x `real(x)`, `imag(x)`
 absolute value, argument of x `abs(x)`, `arg(x)`
 square/ n th root of x `sqrt(x)`, `sqrtn(x, n, {&z})`
 all n -th roots of 1 `rootsof1(n)`
 FFT of $[f_0, \dots, f_{n-1}]$ `w = fftinit(n)`, `fft/fftinw(w, f)`
 trig functions `sin, cos, tan, cotan, sinc`
 inverse trig functions `asin, acos, atan`
 hyperbolic functions `sinh, cosh, tanh, cotanh`
 inverse hyperbolic functions `asinh, acosh, atanh`
 $\log(x)$, $\log(1+x)$, e^x , $e^x - 1$ `log, loglp, exp, expm1`
 Euler Γ function, $\log \Gamma$, Γ'/Γ `gamma, lngamma, psi`
 half-integer gamma function $\Gamma(n+1/2)$ `gammah(n)`
 Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`
 $\sum_{1 \leq n \leq N} n^s$ `dirpowersum(N, s)`
 Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ `zetahurwitz(s, x)`
 Lerch $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$ `lerchphi(z, s, x)`
 Lerch $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$ `lerchzeta(s, x, t)`
 multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ `zetamult(s, {T})`
 all MZVs for weight $\sum s_i = n$ `zetamultall(n)`
 convert MZV id to $[s_1, \dots, s_k]$ `zetamultconvert(f, {flag})`
 MZV dual sequence `zetamultdual(s)`
 multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ `polylogmult(s, z)`

incomplete Γ function ($y = \Gamma(s)$) `incgam(s, x, {y})`
 complementary incomplete Γ `incgamc(s, x)`
 $\int_x^\infty e^{-t} dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ `eint1, erfc`
 elliptic integral of 1st and 2nd kind `ellK(k)`, `ellE(k)`
 dilogarithm of x `dilog(x)`
 m -th polylogarithm of x `polylog(m, x, {flag})`
 U -confluent hypergeometric function `hyperu(a, b, u)`
 Hypergeometric ${}_pF_q(A, B; z)$ `hypergeom(A, B, z)`
 Bessel $J_n(x)$, $J_{n+1/2}(x)$ `besselj(n, x)`, `besseljh(n, x)`
 Bessel I_ν , K_ν , H_ν^1 , H_ν^2 , Y_ν `(bessel)i, k, h1, h2, y`
 k -th zero of $J_\nu(x)$ `besseljzero(nu, {k = 1})`
 k -th zero of $Y_\nu(x)$ `besselyzero(nu, {k = 1})`
 Airy functions $A_i(x)$, $B_i(x)$ `airy(x)`
 Lambert W : x s.t. $xe^x = y$ `lambertw(y)`
 Teichmuller character of p -adic x `teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or
 $(x-a)^{-\alpha}$ singularity `[a, a]`
 exponential decrease $e^{-\alpha|x|}$ `[±∞, a], α > 0`
 ... $\alpha < -1$
 oscillating as $\cos(kx)$ `α = kI, k > 0`
 oscillating as $\sin(kx)$ `α = -kI, k > 0`

numerical integration `intnum(x = a, b, f, {T})`
 weights T for `intnum` `intnuminit(a, b, {m})`
 weights T incl. kernel K `intfuncinit(t = a, b, K, {m})`
 integrate $(2i\pi)^{-1} f$ on circle $|z-a| = R$ `intcirc(x = a, R, f, {T})`

Other integration methods

n -point Gauss-Legendre `intnumgauss(x = a, b, f, {n})`
 weights for n -point Gauss-Legendre `intnumgaussinit({n})`
 quasi-periodic function, period $2H$ `intnumosc(x = a, f, H)`
 Romberg (low accuracy) `intnumromb(x = a, b, f, {flag})`

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy) `suminf(n = a, expr)`
 sum of alternating/positive series `sumalt, sumpos`
 sum of series using Euler-Maclaurin `sumnum(n = a, f, {T})`
 ... Sidi summation `sumnumsidi(n = a, f)`
 $\sum_{n \geq a} F(n)$, F rational function `sumnumrat(F, a)`
 ... $\sum_{p \geq a} F(p^s)$ `sumeulerrat(F, {s = 1}, {a = 2})`
 weights for `sumnum`, a as in DE `sumnuminit({∞, a})`
 sum of series by Monien summation `sumnummonien(n = a, f, {T})`
 weights for `sumnummonien` `sumnummonieninit({∞, a})`
 sum of series using Abel-Plana `sumnumap(n = a, f, {T})`
 weights for `sumnumap`, a as in DE `sumnumapinit({∞, a})`
 sum of series using Lagrange `sumnumlagrange(n = a, f, {T})`
 weights for `sumnumlagrange` `sumnumlagrangeinit`

Products

product $a \leq X \leq b$, initialized at x `prod(X = a, b, expr, {x})`
 product over primes $a \leq X \leq b$ `prodeuler(X = a, b, expr)`
 infinite product $a \leq X \leq \infty$ `prodinf(X = a, expr)`
 $\prod_{n \geq a} F(n)$, F rational function `prodnumrat(F, a)`
 $\prod_{p \geq a} F(p^s)$ `prodeulerrat(F, {s = 1}, {a = 2})`

Other numerical methods

real root of f in $[a, b]$; bracketed root `solve($X = a, b, f$)`
 ... interval splitting, step s `solvestep($X = a, b, s, f, \{flag = 0\}$)`
 limit of $f(t)$, $t \rightarrow \infty$ `limitnum($f, \{\alpha\}$)`
 asymptotic expansion of f (rational) `asymnum($f, \{\alpha\}$)`
 ... $N + 1$ terms as floats `asymnumraw($f, N, \{\alpha\}$)`
 numerical derivation w.r.t x : $f'(a)$ `derivnum($x = a, f$)`
 evaluate continued fraction F at t `contfracval($F, t, \{L\}$)`
 power series to cont. fraction (L terms) `contfracinit($S, \{L\}$)`
 Padé approximant (deg. denom. $\leq B$) `bestapprPade($S, \{B\}$)`

Elementary Arithmetic Functions

vector of binary digits of $|x|$ `binary(x)`
 bit number n of integer x `bittest(x, n)`
 Hamming weight of integer x `hammingweight(x)`
 digits of integer x in base B `digits($x, \{B = 10\}$)`
 sum of digits of integer x in base B `sumdigits($x, \{B = 10\}$)`
 integer from digits `fromdigits($v, \{B = 10\}$)`
 ceiling/floor/fractional part `ceil, floor, frac`
 round x to nearest integer `round($x, \{\&e\}$)`
 truncate x `truncate($x, \{\&e\}$)`
 gcd/LCM of x and y `gcd(x, y), lcm(x, y)`
 gcd of entries of a vector/matrix `content(x)`

Primes and Factorization

extra prime table `addprimes()`
 add primes in v to prime table `addprimes(v)`
 remove primes from prime table `removeprimes(v)`
 Chebyshev $\pi(x)$, n -th prime p_n `primepi(x), prime(n)`
 vector of first n primes `primes(n)`
 smallest prime $\geq x$ `nextprime(x)`
 largest prime $\leq x$ `preprime(x)`
 factorization of x `factor($x, \{lim\}$)`
 ... selecting specific algorithms `factorint($x, \{flag = 0\}$)`
 $n = df^2$, d squarefree/fundamental `core($n, \{fl\}$), coredisc`
 certificate for (prime) N `primecert(N)`
 verifies a certificate c `primecertisvalid(c)`
 convert certificate to Magma/PRIMO `primecertexport`
 recover x from its factorization `factorback($f, \{e\}$)`
 $x \in \mathbf{Z}$, $|x| \leq X$, $\gcd(N, P(x)) \geq N$ `zncoppersmith($P, N, X, \{B\}$)`
 divisors of N in residue class r mod s `divisorslensstra(N, r, s)`

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$ `omega(n), bigomega`
 divisors of n / number of divisors $\tau(n)$ `divisors(n), numdiv`
 sum of (k -th powers of) divisors of n `sigma($n, \{k\}$)`
 Möbius μ -function `moebius(x)`
 Ramanujan's τ -function `ramanujantau(x)`

Combinatorics

factorial of x `x!` or `factorial(x)`
 binomial coefficient $\binom{x}{k}$ `binomial($x, \{k\}$)`
 Bernoulli number B_n as real/rational `bernreal(n), bernfrac`
 $[B_0, B_2, \dots, B_{2k}]$ `bernvec(k)`
 Bernoulli polynomial $B_n(x)$ `bernpol($n, \{x\}$)`
 Euler numbers `eulerfrac, eulerreal, eulervec`
 Euler polynomial $E_n(x)$ `eulerpol($n, \{x\}$)`
 Eulerian polynomial $A_n(x)$ `eulerianpol`
 Fibonacci number F_n `fibonacci(n)`
 Harmonic number $H_{n,r} = 1^{-r} + \dots + n^{-r}$ `harmonic(n, r)`
 Stirling numbers $s(n, k)$ and $S(n, k)$ `stirling($n, k, \{flag\}$)`

Pari-GP reference card

(PARI-GP version 2.16.2)

number of partitions of n `numbpart(n)`
 k -th permutation on n letters `numtoperm(n, k)`
 ... index k of permutation v `permtotnum(v)`
 order of permutation p `permorder(p)`
 signature of permutation p `permsign(p)`
 cyclic decomposition of permutation p `permcycles(p)`
Multiplicative groups ($\mathbf{Z}/N\mathbf{Z}$)*, \mathbf{F}_q^*
 Euler ϕ -function `eulerphi(x)`
 multiplicative order of x (divides o) `znorder($x, \{o\}$), fforder`
 primitive root mod q / x .mod `znprimroot(q), fprimroot(x)`
 structure of $(\mathbf{Z}/n\mathbf{Z})^*$ `znstar(n)`
 discrete logarithm of x in base g `znlog($x, g, \{o\}$), fflag`
 Kronecker-Legendre symbol $(\frac{x}{y})$ `kronecker(x, y)`
 quadratic Hilbert symbol (at p) `hilbert($x, y, \{p\}$)`
Euclidean algorithm, continued fractions
 CRT: solve $z \equiv x$ and $z \equiv y$ `chinese(x, y)`
 minimal u, v so $xu + yv = \gcd(x, y)$ `gcdext(x, y)`
 half-gcd algorithm `halfgcd(x, y)`
 continued fraction of x `contfrac($x, \{b\}, \{lmax\}$)`
 last convergent of continued fraction x `contfracpnqn(x)`
 rational approximation to x (den. $\leq B$) `bestappr($x, \{B\}$)`
 recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$ `bestapprnf(x, T)`

Miscellaneous

integer square / n -th root of x `sqrtnint(x), sqrtint(x, n)`
 largest integer e s.t. $b^e \leq x$, $e = \lfloor \log_b(x) \rfloor$ `logint($x, b, \{\&z\}$)`

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charmul, charconj, chardiv, charpow`
 order of χ `charorder(cyc, χ)`
 kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
 Galois orbits of characters `chargalois(G)`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ `G = znstar($q, 1$)`
 convert datum D to $[G, \chi]$ `znchar(D)`
 is χ odd? `zncharisodd(G, χ)`
 real $\chi \rightarrow$ Kronecker symbol (D/\cdot) `znchartokronecker(G, χ)`
 conductor of χ `zncharconductor(G, χ)`
 $[G_0, \chi_0]$ primitive attached to χ `znchartoprimitive(G, χ)`
 induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ `zncharinduce(G, χ, N)`
 χp `znchardecompose(G, χ, p)`
 $\prod_p |(Q, N) \chi p$ `znchardecompose(G, χ, Q)`
 complex Gauss sum $G_a(\chi)$ `znchargauss(G, χ)`

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character `znconreychar(G, m)`
 character \rightarrow Conrey label `znconreyexp(G, χ)`
 log on Conrey generators `znconreylog(G, m)`
 conductor of χ (χ_0 primitive) `znconreyconductor($G, \chi, \{\chi_0\}$)`

True-False Tests

is x the disc. of a quadratic field? `isfundamental(x)`
 is x a prime? `isprime(x)`
 is x a strong pseudo-prime? `ispseudoprime(x)`
 is x square-free? `issquarefree(x)`
 is x a square? `issquare($x, \{\&n\}$)`
 is x a perfect power? `ispower($x, \{k\}, \{\&n\}$)`
 is x a perfect power of a prime? ($x = p^n$) `isprimepower($x, \&n$)`
 ... of a pseudoprime? `ispseudoprimepower($x, \&n$)`
 is x powerful? `ispowerful(x)`
 is x a totient? ($x = \varphi(n)$) `istotient($x, \{\&n\}$)`
 is x a polygonal number? ($x = P(s, n)$) `ispolygonal($x, s, \{\&n\}$)`
 is pol irreducible? `polisirreducible(pol)`

Graphic Functions

crude graph of $expr$ between a and b `plot($X = a, b, expr$)`
High-resolution plot (immediate plot)
 plot $expr$ between a and b `plotth($X = a, b, expr, \{flag\}, \{n\}$)`
 plot points given by lists lx, ly `plotthraw($lx, ly, \{flag\}$)`
 terminal dimensions `plotsizes()`
Rectwindow functions
 init window w , with size x, y `plotinit(w, x, y)`
 erase window w `plotkill(w)`
 copy w to w_2 with offset (dx, dy) `plotcopy(w, w_2, dx, dy)`
 clips contents of w `plotclip(w)`
 scale coordinates in w `plotscale(w, x_1, x_2, y_1, y_2)`
`plotth` in w `plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)`
`plotthraw` in w `plotrecthraw($w, data, \{flag\}$)`
 draw window w_1 at $(x_1, y_1), \dots$ `plotdraw([[w_1, x_1, y_1], ...])`

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor(w, c)`
 current position of cursor in w `plotcursor(w)`
 write s at cursor's position `plotstring(w, s)`
 move cursor to (x, y) `plotmove(w, x, y)`
 move cursor to $(x + dx, y + dy)$ `plotrmove(w, dx, dy)`
 draw a box to (x_2, y_2) `plotbox(w, x_2, y_2)`
 draw a box to $(x + dx, y + dy)$ `plotrbox(w, dx, dy)`
 draw polygon `plotlines($w, lx, ly, \{flag\}$)`
 draw points `plotpoints(w, lx, ly)`
 draw ellipse `plotell($w, lx, ly, \{flag\}$)`
 draw line to $(x + dx, y + dy)$ `plotrline(w, dx, dy)`
 draw point $(x + dx, y + dy)$ `plotrpoint(w, dx, dy)`

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".
 as `plotth` `plotthexport($f, X = a, b, expr, \{flag\}, \{n\}$)`
 as `plotthraw` `plotthrawexport($f, lx, ly, \{flag\}$)`
 as `plotdraw` `plotexport($f, [[w_1, x_1, y_1], ...])$`

Based on an earlier version by Joseph H. Silverman
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