Algebraic Number Theory

(PARI-GP version 2.18.1)

Binary Quadratic Forms

create $ax^2+bxy+cy^2$	`Qfb`(a,b,c) or `Qfb`$([a,b,c])$
reduce x ($s=\sqrt{D}$, $l=\lfloor s\rfloor$)	`qfbred`$(x,\{flag\},\{D\},\{l\},\{s\})$
return $[y,g]$, $g\in \mathrm{SL}_2(\mathbf{Z})$, $y=g\cdot x$ reduced	`qfbredsl2`(x)
composition of forms	x`*`y or `qfbnucomp`(x,y,l)
n-th power of form	x`^`n or `qfbnupow`(x,n)
composition	`qfbcomp`(x,y)
...without reduction	`qfbcompraw`(x,y)
n-th power	`qfbpow`(x,n)
...without reduction	`qfbpowraw`(x,n)
prime form of disc. x above prime p	`qfbprimeform`(x,p)
class number of disc. x	`qfbclassno`(x)
Hurwitz class number of disc. x	`qfbhclassno`(x)
solve $Q(x,y)=n$ in integers	`qfbsolve`(Q,n)
solve $x^2+Dy^2=p$, p prime	`qfbcornacchia`(D,p)
...$x^2+Dy^2=4p$, p prime	`qfbcornacchia`$(D,4*p)$

Quadratic Fields

quadratic number $\omega=\sqrt{x}$ or $(1+\sqrt{x})/2$	`quadgen`(x)
minimal polynomial of ω	`quadpoly`(x)
discriminant of $\mathbf{Q}(\sqrt{x})$	`quaddisc`(x)
regulator of real quadratic field	`quadregulator`(x)
fundamental unit in O_D, $D>0$	`quadunit`$(D,\{'w\})$
norm of fundamental unit in O_D	`quadunitnorm`(D)
index of $O_{Df^2}^\times$ in $O_D^\times$	`quadunitindex`(D,f)
class group of $\mathbf{Q}(\sqrt{D})$	`quadclassunit`$(D,\{flag\},\{t\})$
Hilbert class field of $\mathbf{Q}(\sqrt{D})$	`quadhilbert`$(D,\{flag\})$
...using specific class invariant ($D<0$)	`polclass`$(D,\{inv\})$
test if T is `polclass`(D); if so return D	`polisclass`(T)
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$	`quadray`$(D,f,\{flag\})$

General Number Fields: Initializations

The number field $K=\mathbf{Q}[X]/(f)$ is given by irreducible $f\in\mathbf{Q}[X]$. We denote $\theta=\bar{X}$ the canonical root of f in K. A *nf* structure contains a maximal order and allows operations on elements and ideals. A *bnf* adds class group and units. A *bnr* is attached to ray class groups and class field theory. A *rnf* is attached to relative extensions L/K.

init number field structure *nf*	`nfinit`$(f,\{flag\})$
known integer basis B	`nfinit`$([f,B])$
order maximal at $vp=[p_1,\dots,p_k]$	`nfinit`$([f,vp])$
order maximal at all $p\le P$	`nfinit`$([f,P])$
certify maximal order	`nfcertify`(nf)
nf members:	
a monic $F\in\mathbf{Z}[X]$ defining K	*nf*`.pol`
number of real/complex places	*nf*`.r1/r2/sign`
discriminant of *nf*	*nf*`.disc`
primes ramified in *nf*	*nf*`.p`
T_2 matrix	*nf*`.t2`
complex roots of F	*nf*`.roots`
integral basis of $\mathbf{Z}_K$ as powers of θ	*nf*`.zk`
different/codifferent	*nf*`.diff`, *nf*`.codiff`
index $[\mathbf{Z}_K:\mathbf{Z}[X]/(F)]$	*nf*`.index`
recompute *nf* using current precision	`nfnewprec`(nf)
init relative *rnf* $L=K[Y]/(g)$	`rnfinit`(nf,g)
init *bnf* structure	`bnfinit`$(f,1)$
bnf members: same as *nf*, plus	
underlying *nf*	*bnf*`.nf`
class group, regulator	*bnf*`.clgp`, *bnf*`.reg`
fundamental/torsion units	*bnf*`.fu`, *bnf*`.tu`
add S-class group and units, yield *bnf*S	`bnfsunit`(bnf,S)
init class field structure *bnr*	`bnrinit`$(bnf,m,\{flag\})$
bnr members: same as *bnf*, plus	
underlying *bnf*	*bnr*`.bnf`
big ideal structure	*bnr*`.bid`
modulus m	*bnr*`.mod`
structure of $(\mathbf{Z}_K/m)^*$	*bnr*`.zkst`

Fields, subfields, embeddings

Defining polynomials, embeddings

(some) number fields with Galois group G	`nflist`(G)
...and $\lvert\mathrm{disc}(K)\rvert=N$ and s complex places	`nflist`$(G,N,\{s\})$
...and $a\le\lvert\mathrm{disc}(K)\rvert\le b$	`nflist`$(G,[a,b],\{s\})$
smallest poly defining $f=0$ (slow)	`polredabs`$(f,\{flag\})$
small poly defining $f=0$ (fast)	`polredbest`$(f,\{flag\})$
monic integral $g=Cf(x/L)$	`poltomonic`$(f,\{\&L\})$
random Tschirnhausen transform of f	`poltschirnhaus`(f)
$\mathbf{Q}[t]/(f)\subset\mathbf{Q}[t]/(g)$? Isomorphic?	`nfisincl`(f,g), `nfisisom`
reverse polmod $a=A(t)$ mod $T(t)$	`modreverse`(a)
compositum of $\mathbf{Q}[t]/(f)$, $\mathbf{Q}[t]/(g)$	`polcompositum`$(f,g,\{flag\})$
compositum of $K[t]/(f)$, $K[t]/(g)$	`nfcompositum`$(nf,f,g,\{flag\})$
splitting field of K (degree divides d)	`nfsplitting`$(nf,\{d\})$
signs of real embeddings of x	`nfeltsign`$(nf,x,\{pl\})$
complex embeddings of x	`nfeltembed`$(nf,x,\{pl\})$
$T\in K[t]$, # of real roots of $\sigma(T)\in R[t]$	`nfpolsturm`$(nf,T,\{pl\})$
absolute Weil height	`nfweilheight`(nf,v)

Subfields, polynomial factorization

subfields (of degree d) of *nf*	`nfsubfields`$(nf,\{d\})$
maximal subfields of *nf*	`nfsubfieldsmax`(nf)
maximal CM subfield of *nf*	`nfsubfieldscm`(nf)
$K_d\subset\mathbf{Q}(\zeta_n)$, using Gaussian periods	`polsubcyclo`$(n,d,\{v\})$
...using class field theory	`polsubcyclofast`(n,d)
roots of unity in *nf*	`nfrootsof1`(nf)
roots of g belonging to *nf*	`nfroots`(nf,g)
factor g in *nf*	`nffactor`(nf,g)

Linear and algebraic relations

poly of degree $\le k$ with root $x\in\mathbf{C}$ or $\mathbf{Q}_p$	`algdep`(x,k)
alg. dep. with pol. coeffs for series s	`seralgdep`(s,x,y)
diff. dep. with pol. coeffs for series s	`serdiffdep`(s,x,y)
small linear rel. on coords of vector x	`lindep`(x)

Basic Number Field Arithmetic (nf)

Number field elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis *nf*`.zk`).

Basic operations

$x+y$	`nfeltadd`(nf,x,y)
$x\times y$	`nfeltmul`(nf,x,y)
x^n, $n\in\mathbf{Z}$	`nfeltpow`(nf,x,n)
x/y	`nfeltdiv`(nf,x,y)
$q=x$`\/`y :=`round`(x/y)	`nfeltdiveuc`(nf,x,y)
$r=x$`%`$y:=x-(x$`\/`$y)y$	`nfeltmod`(nf,x,y)
...$[q,r]$ as above	`nfeltdivrem`(nf,x,y)
reduce x modulo ideal A	`nfeltreduce`(nf,x,A)
absolute trace $\mathrm{Tr}_{K/\mathbf{Q}}(x)$	`nfelttrace`(nf,x)
absolute norm $\mathrm{N}_{K/\mathbf{Q}}(x)$	`nfeltnorm`(nf,x)
is x a square?	`nfeltissquare`$(nf,x,\{\&y\})$
...an n-th power?	`nfeltispower`$(nf,x,n,\{\&y\})$

Multiplicative structure of K^*; $K^*/(K^*)^n$

valuation $v_{\mathfrak{p}}(x)$	`nfeltval`$(nf,x,\mathfrak{p})$
...write $x=\pi^{v_{\mathfrak{p}}(x)}y$	`nfeltval`$(nf,x,\mathfrak{p},\&y)$
quadratic Hilbert symbol (at $\mathfrak{p}$)	`nfhilbert`$(nf,a,b,\{\mathfrak{p}\})$
b such that $xb^n=v$ is small	`idealredmodpower`(nf,x,n)

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$	`nfbasis`(f)
field discriminant of $\mathbf{Q}[x]/(f)$	`nfdisc`(f)
...and factorization	`nfdiscfactors`(f)
express x on integer basis	`nfalgtobasis`(nf,x)
express element x as a polmod	`nfbasistoalg`(nf,x)

Hecke Grossencharacters

Let K be a number field and m a modulus. A gchar structure describes the group of Hecke Grossencharacters of K of modulus m and allows computations with these characters. A character χ is described by its components modulo *gc*`.cyc`.

init gchar structure *gc* for modulus m	`gcharinit`$(bnf,m,\{cm\})$
gc members:	
underlying *bnf*	*gc*`.bnf`
modulus	*gc*`.mod`
elementary divisors (including 0s)	*gc*`.cyc`
recompute *gc* using current precision	`gcharnewprec`(gc)
evaluate Hecke character *chi* at ideal *id*	`gchareval`(gc,chi,id)
exponent column of *id* in $\mathbf{R}^n$	`gcharideallog`(gc,id)
log representation of ideal *id*	`gcharlog`(gc,id)
...of character χ	`gcharduallog`(gc,chi)
exponent vector of χ in $\mathbf{R}^n$	`gcharparameters`(gc,chi)
conductor of χ	`gcharconductor`(gc,chi)
L-function of χ	`lfuncreate`$([gc,chi])$
local component χ_v of χ	`gcharlocal`(gc,chi,v)
χ s.t. $\chi_v\approx Lchiv[i]$ for $v=Lv[i]$	`gcharidentify`$(gc,Lv,Lchiv)$
basis of group of algebraic characters	`gcharalgebraic`(gc)
algebraic character of given infinity type	`gcharalgebraic`$(gc,type)$
is χ algebraic?	`gcharisalgebraic`(gc,chi)

Dedekind Zeta Function ζ_K, Hecke L series

$R=[c,w,h]$ in initialization means we restrict $s\in\mathbf{C}$ to domain $|\Re(s)-c|<w$, $|\Im(s)|<h$; $R=[w,h]$ encodes $[1/2,w,h]$ and $[h]$ encodes $R=[1/2,0,h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I)\le b$	`dirzetak`(nf,b)
init $\zeta_K^{(k)}(s)$ for $k\le n$	`L = lfuninit`$(bnf,R,\{n=0\})$
compute $\zeta_K(s)$ (n-th derivative)	`lfun`$(L,s,\{n=0\})$
compute $\Lambda_K(s)$ (n-th derivative)	`lfunlambda`$(L,s,\{n=0\})$
init $L_K^{(k)}(s,\chi)$ for $k\le n$	`L = lfuninit`$([bnr,chi],R,\{n=0\})$
compute $L_K(s,\chi)$ (n-th derivative)	`lfun`$(L,s,\{n\})$
Artin root number of K	`bnrrootnumber`$(bnr,chi,\{flag\})$
$L(1,\chi)$, for all χ trivial on H	`bnrL1`$(bnr,\{H\},\{flag\})$

Class Groups & Units (bnf, bnr)

Class field theory data $a_1,\{a_2\}$ is usually *bnr* (ray class field), *bnr*, H (congruence subgroup) or *bnr*, χ (character on `bnr.clgp`). Any of these define a unique abelian extension of K.

units / S-units	`bnfunits`$(bnf,\{S\})$
remove GRH assumption from *bnf*	`bnfcertify`(bnf)

expo. of ideal x on class gp	`bnfisprincipal(bnf, x, {flag})`
...on ray class gp	`bnrisprincipal(bnr, x, {flag})`
expo. of x on fund. units	`bnfisunit(bnf, x)`
...on S-units, U is `bnfunits(bnf, S)`	`bnfisunit(bnfs, x, U)`
signs of real embeddings of `bnf.fu`	`bnfsignunit(bnf)`
narrow class group	`bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m	`bnrclassno(bnf, m)`
discriminant of class field	`bnrdisc(a1, {a2})`
ray class numbers, l list of moduli	`bnrclassnolist(bnf, l)`
discriminants of class fields	`bnrdisclist(bnf, l, {arch}, {flag})`
decode output from `bnrdisclist`	`bnfdecodemodule(nf, fa)`
is modulus the conductor?	`bnrisconductor(a1, {a2})`
is class field (bnr, H) Galois over K^G	`bnrisgalois(bnr, G, H)`
action of automorphism on `bnr.gen`	`bnrgaloismatrix(bnr, aut)`
apply `bnrgaloismatrix` M to H	`bnrgaloisapply(bnr, M, H)`
characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$	`bnrchar(bnr, g, {v})`
conductor of character χ	`bnrconductor(bnr, chi)`
conductor of extension	`bnrconductor(a1, {a2}, {flag})`
conductor of extension $K[Y]/(g)$	`rnfconductor(bnf, g)`
canonical projection $\mathrm{Cl}_F \to \mathrm{Cl}_f$, $f \mid F$	`bnrmap`
Artin group of extension $K[Y]/(g)$	`rnfnormgroup(bnr, g)`
subgroups of bnr, index $<= b$	`subgrouplist(bnr, b, {flag})`
compositum as `[bnr,H]`	`bnrcompositum([bnr1, H1], [bnr2, H2])`
class field defined by $H < \mathrm{Cl}_f$	`bnrclassfield(bnr, H)`
...low level equivalent, prime degree	`rnfkummer(bnr, H)`
same, using Stark units (real field)	`bnrstark(bnr, {sub}, {flag})`
Stark unit	`bnrstarkunit(bnr, {sub})`
is a an n-th power in K_v ?	`nfislocalpower(nf, v, a, n)`
cyclic L/K satisf. local conditions	`nfgrunwaldwang(nf, P, D, pl)`

Cyclotomic and Abelian fields theory

An Abelian field F given by a subgroup $H \subset (Z/fZ)^*$ is described by an argument F, e.g. f (for $H = 1$, i.e. $Q(\zeta_f)$) or $[G, H]$, where G is `idealstar(f,1)`, or a minimal polynomial.

minus class number $h^-(F)$	`subcyclohminus(F)`
...p-part	`subcyclohminus(F, p)`
minus part of Iwasawa polynomials	`subcycloiwasawa(F, p)`
p-Sylow of $\mathrm{Cl}(F)$	`subcyclopclgp(F, p)`

Logarithmic class group

logarithmic ℓ-class group	`bnflog(bnf, ℓ)`
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	`bnflogef(bnf, pr)`
$\exp \deg_F(A)$	`bnflogdegree(bnf, A, ℓ)`
is ℓ-extension L/K locally cyclotomic	`rnfislocalcyclo(rnf)`

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	`nfisideal(nf, id)`
is x principal in bnf ?	`bnfisprincipal(bnf, x)`
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	`idealtwoelt(nf, x, {a})`
put ideal a $(a\mathbf{Z}_K + b\mathbf{Z}_K)$ in HNF form	`idealhnf(nf, a, {b})`
norm of ideal x	`idealnorm(nf, x)`
minimum of ideal x (direction v)	`idealmin(nf, x, v)`
LLL-reduce the ideal x (direction v)	`idealred(nf, x, {v})`

Ideal Operations

add ideals x and y	`idealadd(nf, x, y)`
multiply ideals x and y	`idealmul(nf, x, y, {flag})`
intersection of ideal x with Q	`idealdown(nf, x)`
intersection of ideals x and y	`idealintersect(nf, x, y, {flag})`
n-th power of ideal x	`idealpow(nf, x, n, {flag})`
inverse of ideal x	`idealinv(nf, x)`

Algebraic Number Theory

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divide ideal x by y	`idealdiv(nf, x, y, {flag})`
Find $(a, b) \in x \times y$, $a + b = 1$	`idealaddtoone(nf, x, {y})`
coprime integral A, B such that $x = A/B$	`idealnumden(nf, x)`

Primes and Multiplicative Structure

check whether x is a maximal ideal	`idealismaximal(nf, x)`
factor ideal x in $\mathbf{Z}_K$	`idealfactor(nf, x)`
expand ideal factorization in K	`idealfactorback(nf, f, {e})`
is ideal A an n-th power ?	`idealispower(nf, A, n)`
expand elt factorization in K	`nffactorback(nf, f, {e})`
decomposition of prime p in $\mathbf{Z}_K$	`idealprimedec(nf, p)`
valuation of x at prime ideal pr	`idealval(nf, x, pr)`
weak approximation theorem in nf	`idealchinese(nf, x, y)`
$a \in K$, s.t. $v_{\mathfrak{p}}(a) = v_{\mathfrak{p}}(x)$ if $v_{\mathfrak{p}}(x) \neq 0$	`idealappr(nf, x)`
$a \in K$ such that $(a \cdot x, y) = 1$	`idealcoprime(nf, x, y)`
give bid =structure of $(\mathbf{Z}_K/id)^*$	`idealstar(nf, id, {flag})`
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$	`idealprincipalunits(nf, pr, k)`
discrete log of x in $(\mathbf{Z}_K/bid)^*$	`ideallog(nf, x, bid)`
`idealstar` of all ideals of norm $\leq b$	`ideallist(nf, b, {flag})`
add Archimedean places	`ideallistarch(nf, b, {ar}, {flag})`
init `modpr` structure	`nfmodprinit(nf, pr, {v})`
project t to $\mathbf{Z}_K/pr$	`nfmodpr(nf, t, modpr)`
lift from $\mathbf{Z}_K/pr$	`nfmodprlift(nf, t, modpr)`

Galois theory over Q

conjugates of a root θ of nf	`nfgaloisconj(nf, {flag})`
apply Galois automorphism s to x	`nfgaloisapply(nf, s, x)`
Galois group of field $\mathbf{Q}[x]/(f)$	`polgalois(f)`
resolvent field of $\mathbf{Q}[x]/(f)$	`nfresolvent(f)`
initializes a Galois group structure G	`galoisinit(pol, {den})`
...for the splitting field of pol	`galoissplittinginit(pol, {d})`
character table of G	`galoischartable(G)`
conjugacy classes of G	`galoisconjclasses(G)`
$\det(1 - \rho(g)T)$, χ character of ρ	`galoischarpoly(G, χ, {o})`
$\det(\rho(g))$, χ character of ρ	`galoischardet(G, χ, {o})`
action of p in nfgaloisconj form	`galoispermtopol(G, {p})`
identify as abstract group	`galoisidentify(G)`
export a group for GAP/MAGMA	`galoisexport(G, {flag})`
subgroups of the Galois group G	`galoissubgroups(G)`
is subgroup H normal?	`galoisisnormal(G, H)`
subfields from subgroups	`galoissubfields(G, {flag}, {v})`
fixed field	`galoisfixedfield(G, perm, {flag}, {v})`
Frobenius at maximal ideal P	`idealfrobenius(nf, G, P)`
ramification groups at P	`idealramgroups(nf, G, P)`
is G abelian?	`galoisisabelian(G, {flag})`
abelian number fields/$\mathbf{Q}$	`galoissubcyclo(N,H,{flag},{v})`

The `galpol` package

query the package: polynomial	`galoisgetpol(a,b,{s})`
...: permutation group	`galoisgetgroup(a,b)`
...: group description	`galoisgetname(a,b)`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	`rnfequation(nf, T, {flag})`
is L/K abelian?	`rnfisabelian(nf, T)`
relative `nfalgtobasis`	`rnfalgtobasis(rnf, x)`
relative `nfbasistoalg`	`rnfbasistoalg(rnf, x)`
relative `idealhnf`	`rnfidealhnf(rnf, x)`
relative `idealmul`	`rnfidealmul(rnf, x, y)`
relative `idealtwoelt`	`rnfidealtwoelt(rnf, x)`

Lifts and Push-downs

absolute $\to$ relative representation for x	`rnfeltabstorel(rnf, x)`
relative $\to$ absolute representation for x	`rnfeltreltoabs(rnf, x)`
lift x to the relative field	`rnfeltup(rnf, x)`
push x down to the base field	`rnfeltdown(rnf, x)`
idem for x ideal: `(rnfideal)reltoabs`, `abstorel`, `up`, `down`	

Norms and Trace

relative norm of element $x \in L$	`rnfeltnorm(rnf, x)`
relative trace of element $x \in L$	`rnfelttrace(rnf, x)`
absolute norm of ideal x	`rnfidealnormabs(rnf, x)`
relative norm of ideal x	`rnfidealnormrel(rnf, x)`
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	`bnfisintnorm(bnf, x)`
is $x \in \mathbf{Q}$ a norm from K?	`bnfisnorm(bnf, x, {flag})`
initialize T for norm eq. solver	`rnfisnorminit(K, pol, {flag})`
is $a \in K$ a norm from L?	`rnfisnorm(T, a, {flag})`
initialize t for Thue equation solver	`thueinit(f)`
solve Thue equation $f(x, y) = a$	`thue(t, a, {sol})`
characteristic poly. of a mod T	`rnfcharpoly(nf, T, a, {v})`

Factorization

factor ideal x in L	`rnfidealfactor(rnf, x)`
$[S, T]$: $T_{i,j} \mid S_i$; S primes of K above p	`rnfidealprimedec(rnf, p)`

Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$-module

relative `polredbest`	`rnfpolredbest(nf, T)`
relative `polredabs`	`rnfpolredabs(nf, T)`
relative Dedekind criterion, prime pr	`rnfdedekind(nf, T, pr)`
discriminant of relative extension	`rnfdisc(nf, T)`
pseudo-basis of $\mathbf{Z}_L$	`rnfpseudobasis(nf, T)`

General $\mathbf{Z}_K$-modules: $M = [\text{matrix}, \text{vec. of ideals}] \subset L$

relative HNF / SNF	`nfhnf(nf, M)`, `nfsnf`
multiple of det M	`nfdetint(nf, M)`
HNF of M where $d = nfdetint(M)$	`nfhnfmod(x, d)`
reduced basis for M	`rnflllgram(nf, T, M)`
determinant of pseudo-matrix M	`rnfdet(nf, M)`
Steinitz class of M	`rnfsteinitz(nf, M)`
$\mathbf{Z}_K$-basis of M if $\mathbf{Z}_K$-free, or 0	`rnfhnfbasis(bnf, M)`
n-basis of M, or $(n + 1)$-generating set	`rnfbasis(bnf, M)`
is M a free $\mathbf{Z}_K$-module?	`rnfisfree(bnf, M)`

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Associative Algebras

A is a general associative algebra given by a multiplication table mt (over $\mathbf{Q}$ or $\mathbf{F}_p$); represented by al from `algtableinit`.

create al from mt (over $\mathbf{F}_p$)	`algtableinit`$(mt, \{p=0\})$
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	`alggroup`$(G, \{p=0\})$
center of group algebra	`alggroupcenter`$(G, \{p=0\})$

Properties

is (mt, p) OK for algtableinit?	`algisassociative`$(mt, \{p=0\})$
multiplication table mt	`algmultable`(al)
dimension of A over prime subfield	`algdim`(al)
characteristic of A	`algchar`(al)
is A commutative?	`algiscommutative`(al)
is A simple?	`algissimple`(al)
is A semi-simple?	`algissemisimple`(al)
center of A	`algcenter`(al)
Jacobson radical of A	`algradical`(al)
radical J and simple factors of A/J	`algsimpledec`(al)

Operations on algebras

create A/I, I two-sided ideal	`algquotient`(al, I)
create $A_1 \otimes A_2$	`algtensor`$(al1, al2)$
create subalgebra from basis B	`algsubalg`(al, B)
quotients by ortho. central idempotents e	`algcentralproj`(al, e)
isomorphic alg. with integral mult. table	`algmakeintegral`(mt)
prime subalgebra of semi-simple A over $\mathbf{F}_p$	`algprimesubalg`(al)
find isomorphism $A \cong M_d(\mathbf{F}_q)$	`algsplit`(al)

Operations on lattices in algebras

lattice generated by cols. of M	`alglathnf`(al, M)
... by the products xy, $x \in lat1$, $y \in lat2$	`alglatmul`$(al, lat1, lat2)$
sum $lat1 + lat2$ of the lattices	`alglatadd`$(al, lat1, lat2)$
intersection $lat1 \cap lat2$	`alglatinter`$(al, lat1, lat2)$
test $lat1 \subset lat2$	`alglatsubset`$(al, lat1, lat2)$
generalized index $(lat2 : lat1)$	`alglatindex`$(al, lat1, lat2)$
$\{x \in al \mid x \cdot lat1 \subset lat2\}$	`alglatlefttransporter`$(al, lat1, lat2)$
$\{x \in al \mid lat1 \cdot x \subset lat2\}$	`alglatrighttransporter`$(al, lat1, lat2)$
test $x \in lat$ (set c = coord. of x)	`alglatcontains`$(al, lat, x, \{\&c\})$
element of lat with coordinates c	`alglatelement`(al, lat, c)

Operations on elements

$a+b$, $a-b$, $-a$	`algadd`(al, a, b), `algsub`, `algneg`
$a \times b$, a^2	`algmul`(al, a, b), `algsqr`
a^n, a^{-1}	`algpow`(al, a, n), `alginv`
is x invertible ? (then set $z = x^{-1}$)	`algisinv`$(al, x, \{\&z\})$
find z such that $x \times z = y$	`algdivl`(al, x, y)
find z such that $z \times x = y$	`algdivr`(al, x, y)
does z s.t. $x \times z = y$ exist? (set it)	`algisdivl`$(al, x, y, \{\&z\})$
matrix of $v \mapsto x \cdot v$	`algtomatrix`(al, x)
absolute norm	`algnorm`(al, x)
absolute trace	`algtrace`(al, x)
absolute char. polynomial	`algcharpoly`(al, x)
given $a \in A$ and polynomial T, return $T(a)$	`algpoleval`(al, T, a)
random element in a box	`algrandom`(al, b)

Central Simple Algebras

A is a central simple algebra over a number field K; represented by al from `alginit`; K is given by a nf structure.

create CSA from data	`alginit`$(B, C, \{v\}, \{flag=3\})$
multiplication table over K	$B = K$, $C = mt$
cyclic algebra $(L/K, \sigma, b)$	$B = rnf$, $C = [sigma, b]$
quaternion algebra $(a,b)_K$	$B = K$, $C = [a, b]$
matrix algebra $M_d(K)$	$B = K$, $C = d$
local Hasse invariants over K	$B = K$, $C = [d, [PR, HF], HI]$
ramification set of quaternion algebra	$B = K$, $C = [PR, HI]$
Hamilton quaternion algebra $(-1,-1)_{\mathbf{R}}$	$B = 1.$, $C = 1/2$
recover (a,b) if $al = (a,b)_K$	`algisquatalg`(al)
recompute al using current precision	`algnewprec`(al)

Properties

type of al (mt, CSA)	`algtype`(al)
dimension of A over $\mathbf{Q}$	`algdim`$(al, 1)$
dimension of al over its center K	`algdim`(al)
degree of A $(= \sqrt{\dim_K A})$	`algdegree`(al)
al a cyclic algebra $(L/K, \sigma, b)$; return σ	`algaut`(al)
... return b	`algb`(al)
... return L/K, as an rnf	`algsplittingfield`(al)
split A over an extension of K	`algsplittingdata`(al)
splitting field of A as an rnf over center	`algsplittingfield`(al)
multiplication table over center	`algrelmultable`(al)
places of K at which A ramifies	`algramifiedplaces`(al)
Hasse invariants at finite places of K	`alghassef`(al)
Hasse invariants at infinite places of K	`alghassei`(al)
Hasse invariant at place v	`alghasse`(al, v)
index of A over K (at place v)	`algindex`$(al, \{v\})$
is al a division algebra? (at place v)	`algisdivision`$(al, \{v\})$
is A ramified? (at place v)	`algisramified`$(al, \{v\})$
is A split? (at place v)	`algissplit`$(al, \{v\})$
is $A \cong A_2$? (at place v)	`algisisom`$(al, al2, \{v\})$

Operations on elements

$x \in nf$ as an element of al	`algeltfromnf`(al, x)
reduced norm	`algnorm`(al, x)
reduced trace	`algtrace`(al, x)
reduced char. polynomial	`algcharpoly`(al, x)
express x on integral basis	`algalgtobasis`(al, x)
convert x to algebraic form	`algbasistoalg`(al, x)
express quaternion x on integral basis	`algquattobasis`(al, x)
convert x to quaternion form	`algbasistoquat`(al, x)
quaternion conjugate of x	`alginvol`(al, x)
map $x \in A$ to $M_d(L)$, L split. field	`algtomatrix`(al, x)
init mod pr map $\mathcal{O}_0 \to M_k(\mathbf{F}_q)$	`algmodprinit`$(al, pr, \{v\})$
project x to $M_k(\mathbf{F}_q)$	`algmodpr`$(al, x, modP)$
lift from $M_k(\mathbf{F}_q)$ to $\mathcal{O}_0$	`algmodprlift`$(al, x, modP)$
$g \in A^\times$ s.t. $fa = gag^{-1}$	`algskolemnoether`(al, a, fa)

Orders

$\mathbf{Z}$-basis of order $\mathcal{O}_0$	`algbasis`(al)
discriminant of order $\mathcal{O}_0$	`algdisc`(al)
$\mathbf{Z}$-basis of natural order in terms $\mathcal{O}_0$'s basis	`alginvbasis`(al)
$\mathbf{Z}$-basis of Eichler order of level N	`algeichlerbasis`(al, N)

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